

# Generalization of the Dick Model

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## Abstract

We discuss a model with a massless scalar coupling to the Yang-Mills SU(2) gauge field in four-dimensional space-time. The solutions from static, pointlike colour source are given. There exists not only solutions with finite energy but also singular one which describes confinement. The confining potential depends on the  $\delta$  parameter of our model. The regular magnetic monopole solutions as well as the singular dyon configurations are also obtained. We fit the  $\delta$  parameter to the experimental data.

## 1 The model

Recently it has been pointed that in massless [1] and massive [2] scalar field theories with a dilaton coupling to a gauge field exist confining solutions. The potential of the gauge field from static, pointlike colour source is in this case singular in the spatial infinity and grows linearly as  $r \rightarrow \infty$ . However, due to some phenomenological [3], [4] as well as theoretical arguments [5] one may also expect that the potential of confinement is not linear. In our work we discuss a class of models which produces relatively wide spectrum of confining potentials.

In the present paper we would like to focus on a model described by the action:

$$S = \int d^4x \left[ -\frac{1}{4} \left( \frac{\phi}{\Lambda} \right)^{8\delta} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right], \quad (1)$$

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where  $\delta > 0$ ,  $\Lambda$  is a dimensional constant,  $\phi$  is massless scalar field coupled to the gauge field  $A_\mu^a$  and  $F_{\mu\nu}^a$  is defined in the standard manner. The action we consider emerges from Dick model [2] with more general coupling between the scalar and the gauge fields. In the contradistinction to the Dick model our scalar field is massless and there are not simple connections between solutions of these models. One can add a potential or a mass term to the action but as far as we do not know how to choose the ground state for the scalar field it seems better and more general to consider the model without potential. Because of the fact that we are interested in the long range behaviour of the fields i.e. in the small energy limit, we neglect the denominator which is present in the original coupling. However, it is possible to consider the full model with the denominator. Then the solutions will contain a well-know part corresponding to the standard Yang-Mills equations.

The field equations for (1) take the form:

$$D_\mu \left( \frac{\phi}{\Lambda} \right)^{8\delta} F^{a\mu\nu} = j^{a\nu}, \quad (2)$$

$$\partial_\mu \partial^\mu \phi = -2\delta F_{\mu\nu}^a F^{a\mu\nu} \frac{\phi^{8\delta-1}}{\Lambda^{8\delta}}, \quad (3)$$

where  $j^{a\mu}$  is the external colour current density.

## 2 The electric sector

In this section we discuss the Coulomb solution in our model. In the other words we find the solution of the field equations generated by static, pointlike colour source:

$$j^{a\mu} = q\delta(r)C^a\delta^{\mu 0}, \quad (4)$$

where  $1 \leq a \leq N_c^2 - 1$  is an  $\text{su}(N_c)$  Lie algebra index and  $C^a$  is the expectation value of the  $\text{su}(N_c)$  generator for a normalized spinor in colour space (see eg. [1]). The field equations (2), (3) take then the form:

$$\left[ r^2 \left( \frac{\phi}{\Lambda} \right)^{8\delta} E^a \right]' = qC^a\delta(r), \quad (5)$$

$$\nabla_r^2 \phi = -4\delta E_a^2 \frac{\phi^{8\delta-1}}{\Lambda^{8\delta}}, \quad (6)$$

where we use the standard definition  $E^{ai} = -F^{a0i}$ . Eqs. (5), (6) have the following solutions parametrized by  $\beta_0 > 0$ :

$$\phi = A\Lambda \left( \frac{1}{\Lambda r} + \frac{1}{\beta_0} \right)^{\frac{1}{1+4\delta}}, \quad (7)$$

$$E(r) = A^{-8\delta} \frac{q}{r^2} \left( \frac{1}{\Lambda r} + \frac{1}{\beta_0} \right)^{-\frac{8\delta}{1+4\delta}}, \quad (8)$$

where  $A = [q(1 + 4\delta)]^{\frac{1}{1+4\delta}}$  and  $\vec{E}^a(r) = C^a E(r) \hat{r}$ .

One can define additional numbers for the scalar and electric field in the standard manner (see [1],[10]). They are given in the following form:

$$Q = r^2 E(r)|_{r \rightarrow \infty} = A^{-8\delta} q \beta_0^{\frac{8\delta}{1+4\delta}}, \quad (9)$$

$$D = -r^2 \frac{d\phi}{dr} |_{r \rightarrow \infty} = \frac{A}{1 + 4\delta} \beta_0^{\frac{4\delta}{1+4\delta}}. \quad (10)$$

It is possible to regard the numbers as some effective charges of the scalar and electric field. These numbers are not independent and they satisfy the following condition:

$$\frac{D^2}{Q} = q \quad (11)$$

The values of  $D$  and  $Q$  are finite for each, finite  $\beta_0$ . The existence of the effective charges is rather mysterious but it can be understood from symmetries point of view. Namely, using equation (5) we can eliminate the electric field in (6). It is easy to notice that after rewriting this equation in terms of the variable  $x = \frac{1}{r}$  a new symmetry appears. This symmetry is the translational symmetry of the variable  $x$ ,

$$x \longrightarrow x' = x + x_0 \quad \phi(x) \longrightarrow \phi'(x') = \phi(x). \quad (12)$$

The generator of the symmetry has form:  $\hat{D}_x = -\frac{d}{dx}$ , or, in the old variable  $r$ :

$$\hat{D}_r = r^2 \frac{d}{dr} \quad (13)$$

In the natural way we define the effective scalar charge using the generator (13),  $D = -\hat{D}_r \phi(r) |_{r \rightarrow \infty}$  which means that the scalar charge emerges from the symmetry (12). It is possible to break the symmetry by adding same new terms to the action. In the simplest case one can introduce a potential for the scalar field. Moreover, this potential fixes asymptotic behaviour of the scalar field i.e. the value of the parameter  $\beta_0$  as well as the effective scalar charge. However, in our work we do not break the symmetry and continuous spectrum of the solution survives.

The energy density for the solutions (7), (8) takes the form:

$$\varepsilon = A^{-8\delta} \frac{q^2}{r^4} \left( \frac{1}{\Lambda r} + \frac{1}{\beta_0} \right)^{-\frac{8\delta}{1+4\delta}}. \quad (14)$$

Thus we may conclude that the energy of configurations (7) - (8) is finite for  $0 < \beta_0 < \infty$  only if the parameter  $\delta > \frac{1}{4}$ . In this case one finds:

$$E_N = \int \varepsilon r^2 dr = \Lambda \frac{4\delta + 1}{4\delta - 1} A^{-8\delta} q^2 \beta_0^{\frac{4\delta-1}{4\delta+1}}. \quad (15)$$

From the full spectrum of Coulomb solutions the family with finite energy and charges (9), (10) can be then separated. As the energy depends on the parameter  $\beta_0$  it can achieve any positive value. On the other hand there exists the singular solution:

$$\phi(r) = A\Lambda \left( \frac{1}{\Lambda r} \right)^{\frac{1}{1+4\delta}}, \quad (16)$$

$$E(r) = A^{-8\delta} q \Lambda^2 \left( \frac{1}{\Lambda r} \right)^{\frac{2}{1+4\delta}}, \quad (17)$$

which describes the confining sector of our theory. The colour-electric potential has the following form:

$$V(r) = \begin{cases} \frac{4\delta+1}{4\delta-1} A^{-8\delta} q \Lambda^{\frac{4\delta}{4\delta+1}} \cdot r^{\frac{4\delta-1}{4\delta+1}} & \delta > \frac{1}{4} \\ \Lambda A^{-8\delta} q \ln \Lambda r & \delta = \frac{1}{4} \end{cases} \quad (18)$$

The asymptotic behaviour of  $V(r)$  changes from  $\log r$  for  $\delta = \frac{1}{4}$  to the linear potential in the limit  $\delta \rightarrow \infty$ . Obviously, the limit  $\delta \rightarrow \infty$  cannot be implemented on the Lagrangian level. It is not possible to realize the linear flux-tube but it may be approximated with arbitrary accuracy taking sufficiently large value of  $\delta$ . The energy density for the singular solution may be obtained from (114) in the limit  $\beta_0 \rightarrow \infty$ . It should be stressed that on the contrary to the standard Coulomb potential the singularity of energy and charges is caused by the behaviour in the spatial infinity. In fact the energy density is singular at  $r = 0$  but this singularity is integrable for  $\delta > \frac{1}{4}$ .

Elimination of single charge states from the physical spectrum theory is not sufficient to have confinement – like sector in our model. One have to check that a state with two, opposite colour charges, has a finite energy. In that case we shall consider a dipole-like external charge,

$$j^{a\mu} = q \left[ \delta(z + \frac{a}{2}) - \delta(z - \frac{a}{2}) \right] \delta(x) \delta(y) \delta^{a3} \delta^{\mu 0}. \quad (19)$$

We restrict consideration to Abelian sector of the theory. Because of nonlinearity of the equations of motion we are not able to solve them in analytically.

However, as it was presented in [6], [7], [9], some numerical methods can be applied.

It is convenient to define the dual potential  $\vec{C}$ :

$$\vec{D} = \nabla \times \vec{C}, \quad (20)$$

where  $\vec{D} = (\frac{\phi}{\Lambda})^{8\delta} \vec{E}$  is the dielectric induction. After introduction of the cylindrical coordinates  $(\rho, \alpha, z)$  we assume that the dual potential has the form:

$$\vec{C} = \frac{\hat{\alpha}}{2\pi\rho} \Phi(\rho, z), \quad (21)$$

where  $\hat{\alpha}$  is the unit vector tangent to the  $\alpha$  coordinate line and  $\Phi$  is a scalar flux function. Then the equation of motion can be rewritten as:

$$\nabla \left( \frac{1}{\rho} \left( \frac{\phi}{\Lambda} \right)^{-8\delta} \nabla \Phi \right) = 0, \quad (22)$$

$$\nabla^2 \phi + 4\delta \left( \frac{\phi}{\Lambda} \right)^{-8\delta} \frac{\phi}{\rho} |\nabla \Phi|^2 = 0. \quad (23)$$

Following [6], [7] we fix boundary conditions as:

$$\begin{aligned} \Phi &= 0 & \rho &= 0, \quad |z| > R/2 \\ \Phi &= q & \rho &= 0, \quad |z| < R/2. \end{aligned} \quad (24)$$

Moreover, we put that  $\Phi \rightarrow 0, \phi \rightarrow 0$  for  $\rho^2 + z^2 \rightarrow \infty$ . This set of equations can be solved numerically. Fig. 1 presents the flux function  $\Phi$  computed for  $900 \times 900$  mesh,  $\delta = 0.75$  and  $q = 1.1$  (for detailed description of the applied numerical procedure see [7], [9]).

It is possible to construct examples of the flux and the scalar function which obey the boundary condition and have finite energy:

$$\Phi = \frac{q}{2} \left( \frac{z + R/2}{\sqrt{\rho^2 + (z + R/2)^2}} - \frac{z - R/2}{\sqrt{\rho^2 + (z - R/2)^2}} \right), \quad (25)$$

$$\phi = A\Lambda \left( \frac{1}{\Lambda\sqrt{\rho^2 + (z + R/2)^2}} \right)^{\frac{1}{1+4\delta}} - A\Lambda \left( \frac{1}{\Lambda\sqrt{\rho^2 - (z - R/2)^2}} \right)^{\frac{1}{1+4\delta}}. \quad (26)$$

As it was mentioned in [9], in spite the fact that these functions do not obey Eqs. (22), (23) they give an upper bound for the total field energy for the charges  $q, -q$ . One can check that

$$E_N \sim R^{\frac{4\delta-1}{4\delta+1}}. \quad (27)$$

### 3 The magnetic sector

Let us now consider purely magnetic, non-abelian content of our model. We use typical, spherically-symmetric Ansatz:

$$A_i^a = \epsilon_{aik} \frac{x^k}{r^2} (g - 1), \quad A_0^a = 0, \quad (28)$$

where  $g(r)$  is a function of radial coordinate only. Inserting the Ansatz (28) into Eqs. (2), (3) we get:

$$[\phi^{8\delta} g']' + \frac{\phi^{8\delta}}{r^2} g (1 - g^2) = 0, \quad (29)$$

$$-\frac{1}{r^2} (r^2 \phi')' + 4\delta \frac{\phi^{8\delta-1}}{\Lambda^{8\delta}} \left[ \frac{2g'^2}{r^2} + \frac{(g^2 - 1)^2}{r^4} \right] = 0. \quad (30)$$

The vacuum state corresponds to  $\phi = 0$  and arbitrary value of the gauge field or to  $g = \pm 1$  and  $\phi = \phi_0$ , where  $\phi_0$  is an arbitrary constant. The standard magnetic monopole solution may be constructed by taking  $g = 0$ . Then the scalar field is given by a family of configurations parametrized by  $\beta_0$ :

$$\phi(r) = B\Lambda \left( \frac{1}{\Lambda r} + \frac{1}{\beta_0} \right)^{\frac{1}{1-4\delta}}, \quad (31)$$

where  $B = (1 - 4\delta)^{\frac{1}{1-4\delta}}$ . The energy density reads:

$$\varepsilon = \frac{B^2}{(1 - 4\delta)^2} \frac{1}{r^4} \left( \frac{1}{\Lambda r} + \frac{1}{\beta_0} \right)^{\frac{8\delta}{1-4\delta}}, \quad (32)$$

It is easy to check that for each  $0 < \beta_0 < \infty$  the energy for such a configuration is finite.

$$E_N = \Lambda \frac{4\delta - 1}{4\delta + 1} B^{8\delta} q^2 \beta_0^{\frac{4\delta+1}{4\delta-1}}. \quad (33)$$

The effective scalar charge takes the value:

$$D = \frac{B}{1 - 4\delta} \beta_0^{\frac{-4\delta}{1-4\delta}}. \quad (34)$$

Of course there exist also the infinity energy solution, namely:

$$\phi(r) = B\Lambda \left( \frac{1}{\Lambda r} \right)^{\frac{1}{1-4\delta}} \quad (35)$$

In the magnetic case, for  $\delta \geq \frac{1}{4}$ , the asymptotic divergence of the singular solution is much bigger than in the electric case. Because of that magnetic

singular monopole can not form finite energy bound states. The electric Coulomb sector as well as the magnetic one are related to each other by the duality transformation [10]:

$$F_{\mu\nu}^a \rightarrow \phi^{8\delta} F_{\mu\nu}^{*a}, \quad \delta \rightarrow -\delta \quad (36)$$

## 4 The dyon

Let us now consider generalization of Ansatz (28) with electric field:

$$A_0^a = \frac{x^a}{r} h \quad (37)$$

The field equations take then the form:

$$[\phi^{8\delta} g']' = \frac{\phi^{8\delta}}{r^2} [g(g^2 - 1) + r^2 h^2 g] \quad (38)$$

$$[r^2 \phi^{8\delta} h']' = 2\phi^{8\delta} h g^2 \quad (39)$$

$$-\frac{1}{r^2} (r^2 \phi')' + 4\delta \frac{\phi^{8\delta-1}}{\Lambda^{8\delta}} \left[ \frac{2g'^2}{r^2} + \frac{(g^2 - 1)^2}{r^4} - 2\frac{h^2 g^2}{r^2} - h'^2 \right] = 0 \quad (40)$$

The dyon solution is given by the formulae:

$$g = 0, h' = C \left( \frac{\phi}{\Lambda} \right)^{-8\delta} \frac{1}{r^2}, \quad (41)$$

where  $G$  is an arbitrary constant and  $\phi$  satisfies the equation:

$$-\frac{1}{r^2} (r^2 \phi')' + 4\delta \frac{\phi^{8\delta-1}}{\Lambda^{8\delta}} \left[ \frac{1}{r^4} - G^2 \left( \frac{\phi}{\Lambda} \right)^{-16\delta} \frac{1}{r^4} \right] = 0. \quad (42)$$

The dyon solution can be interpreted as standard magnetic monopole surrounded by the region of colour electric field. From Eq. (42) one easily gets:

$$\pm \frac{d\phi}{dx} = \phi^{4\delta} - G\phi^{-4\delta}. \quad (43)$$

There exist only a few values of parameter  $\delta$  for which Eq. (43) can be solved analytically. Otherwise only numerical estimations are known. For example choosing  $\delta = \frac{1}{4}$  the solution reads:

$$\phi(r) = \Lambda \sqrt{G + (g_0 - 1)e^{\mp \frac{2}{\Lambda r}}}. \quad (44)$$

Here  $g_0$  is a constant and the charges are

$$D = \frac{-g_0}{\sqrt{G+g_0}}, \quad Q = \frac{G}{G+g_0}. \quad (45)$$

Analogously for  $\delta = \frac{1}{2}$  one gets:

$$\arctan \frac{\phi}{\Lambda} - \operatorname{arcth} \frac{\phi}{\Lambda} = 2 \left( \frac{1}{\Lambda r} + \frac{1}{\beta_0} \right). \quad (46)$$

It is easy to show that energy for all dyon solutions is infinite. Unfortunately, singularity of the energy emerges for  $r = 0$  and there is no chance for two-dyons, finite energy solutions.

## 5 Conclusions

The main result of our work is that the electric singular solutions can form the dipole-like finite energy states i.e. something like a confining sector of the model emerges: the static solutions with non-vanishing total charge are excluded from the physical spectrum while the  $q, -q$  dipoles can appear. For the discussed model one can find, in the Coulomb sector of the theory, the potential of confinement (18). The behaviour of the energy depends on the model parameter  $\delta$ . Taking into account the form of confining potential which is suggested by phenomenological arguments (see eg. [3]) the parameter  $\delta$  can be fixed. For  $V(r) \sim \sqrt{r}$  one finds  $\delta = \frac{3}{4}$ . On the other hand, there is continuous spectrum of finite energy solutions. Because of the fact that these solutions describe a source with a fixed charge it can be possible to look at them as screening (see eg. [11], [12]).

For the magnetic, sourceless part of the model, it is also possible to separate the finite as well as the infinite energy sector. One can interpret spherically-symmetric, sourceless solutions with finite energy, which form the finite energy sector, as classical glueballs [13]. On the contrary to electric part the magnetic singular monopoles can not form finite energy bound states. So in the sense, magnetic part of the theory does not provide the confinement.

The confinement sector has its equivalent in the Pagels-Toumboulis model [5], [9]. Our scalar field plays the same role as the dielectric function in the effective action [5]. Of course both models are not identical. For example there is not (probably) any screening sector in Pagels-Toumboulis model.

As we see the model (1) posses two phases: confinement and screening (or glueball)-like. The phases refer to different, asymptotic values of the scalar field, zero or non-zero respectively. The asymptotic value of the scalar



field can be fixed by a potential term. Then the confinement phase could be understood as the symmetric phase whereas the screening sector as the phase with broken symmetry. Unfortunately, we do not know the form of the potential. However, color dielectric models [14] can give some hints [15].

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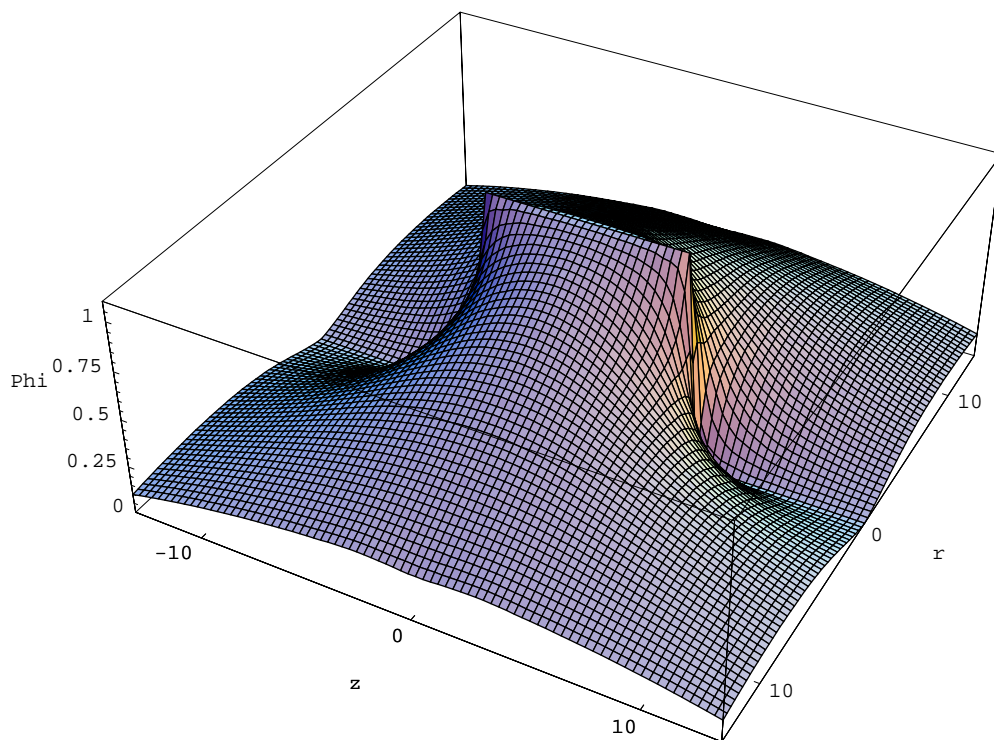


Figure 1: The flux function  $\Phi$  for  $\delta = 0.75$  and  $R = 10$